In our Time Programme 41
Mathematics and Storytelling

Melvyn Bragg : Hello. Today I speak to John Allen Paulos and Marina Warner to examine the links between mathematics and story telling. According to anthropologists they have similar origins in our prehistoric ancestors need to measure and assess the world around them. Both mathematics and stories need a shape and structure to make any sense, but does it go any further than that? Is it possible to apply mathematical logic to literature, or to reduce a joke to an algebraic equation? Or are the literary imagination and scientific substance irreconcilable?

John Allen Paulos is Presidential Scholar of Mathematics at Temple University, Philadelphia. He's the author of "Innumeracy:Mathematical Illiteracy and its Consequences", which became an instant best seller and was subsequently translated into eleven languages. Described as "America's favourite mathematician", his latest book is called "Once upon a number:The Hidden Mathematical Logic of Stories", which look sat the relationships which tie the realm of mathematics and the world of stories together. The novelist, historian and critic Marina Warner is less sure of the links and similarities between mathematics and story telling. A former Reith lecturer, and currently a visiting professor at Birckbeck College, London, her latest book "No Go the Bogeyman", was a study of fear, and she's a special interest in legends, symbolism and story telling.

John Allen Paulos, just....could you give us the gist if it? What d'you think that maths and story telling have in common and what do they contribute to each other?

John Allen Paulos : Well, the complicate..... the relationship's a complicated one. One point that can be made is that stories provide a context for mathematical ideas and applications. Almost any mathematical notion that's introduced can be placed into a story, a little vignette that clarifies it.

Melvyn Bragg : Such as?

John Allen Paulos : Er... er... random walk, if you're talking about random processes, the drunkards random walk or exponential growth, you put a grain of sand on the first square of the checkerboard, two on the second, four on the third and so on, er.. and so on, and it's two to the sixty-fourth on the last one. So er....also these stories or vignettes, kind of expose the limitations of mathematical notions, I mean one and one is two, but if you take a cup of popcorn and a cup of water and add them together, you only get about a cup and half of soggy popcorn, so it's not that... the mathematics is fine, this particular application isn't. Errrm... more generally, I mean stories... er... er the narrative development of mathematical ideas and theorems is important. Too often in mathematics education, there's a theorem, there's (indistinct) Theorem, there's Godel's Theorem , Central Limit Theorem, whatever and they just kind of come out of the woodwork and nobody knows the story, the development, the relevance of it to other notions in mathematics and physics, and going in the other direction, ...er mathematical notions help clarify the structure of stories... er probabilistic notions, notions from information theory, recursive function theory, help us get a grip on stories, and moreover if you look at new stories, often the statistics or whatever provide some muscle for the story.

Melvyn Bragg : Now there's a lot there.....

John Allen Paulos : There's a lot there, yeah.

Melvyn Bragg : ... just a second, we're going to have to, sort of, try to decode that, just.....are you saying that.....? I can see that theorems are like - enough for this conversation - stories, but are you saying that inside mathematics itself, starting with 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, there is a strict logical similarity to the telling of a story, "Once upon a time"? You call it "Once upon a number".

John Allen Paulos : "Once upon a number".

Melvyn Bragg : So once upon a time a little girl or a little boy lived in the middle of a dark wood....

John Allen Paulos : Right.

Melvyn Bragg : ... now is that similar to 1, 2, 3, ... ? What is that similar to in mathematics? How is.....you.....how is the logic, which is the word you use, how is the logic similar?

John Allen Paulos : Well I think that the logic is quite different in a way. I mean mathematical logic for example, is extentional, which means it allows substitutions. That's er....if you have a three in a mathematical computation, you can replace it with the square root of nine, and nothing changes, but er....that substitutability isn't true in narrative logic, intentional logic. It's kind of a nebulous.....

Melvyn Bragg : Yes, but I'm interested at the moment in the similarities....

John Allen Paulos : Okay.

Melvyn Bragg : So, one more crack and then I'll turn to Marina. You say their origins are similar, can you just give us some idea of the similar.....we can talk about differences in a minute....

John Allen Paulos : Right, okay.

Melvyn Bragg : What about the similarity of their origin?

John Allen Paulos : Well, I mean I think the development of stories and of mathematics were kind of prosaic. I mean er....mathematical counting led to formal arithmetic. Measuring fields and so on, led to Greek geometry and in something like the same way I would imagine, stories developed out of more primitive practical sorts of communications, like "hey there's a behemoth over there" or "come here" or "how did you get that fire started"! And then gradually these things were strung together into stories and then into literature, in the same way that Greek formal mathematics, I mean was a er..... development, refinement of these every day rules of thumb that preceded them.

Melvyn Bragg : Marina Warner do you see the similarity in origination between stories and mathematics? Are you convinced that.....or d'you....have you any evidence.....is there any evidence that the one preceded the other?

Marina Warner : Well there's certainly a relationship in language, because you get, in English for example, you get the relationship between "teller" like bank teller, which is counting, which is like tallying , and the idea of course of telling a tale. So that....and....also, there's a very deep symbolic tradition of taking and weighing and measuring the life of a person, so that....which actually goes through of course to St Michael the archangel counting the scales at the last judgement, but goes back to ancient Egypt where your soul was weighed in the pan, at death and seen if you were light or heavy, and you had to be light, you had to weigh against the feather of justice, and that was the "sum", I mean a mathematical idea, the sum of the deeds and your character. So very deep down there is this, as it were, this metaphorical idea of accumulation and of counting and of taking the you know, taking the tally of someone. But....

John Allen Paulos : If I can just interrupt? I mean in addition to teller, I mean words like "account". An account is a story as well.

Marina Warner : Mmm, exactly, yes, yes. But I think that it's ....what is more difficult is the point's actually that you were making more earlier, and that is that there is some kind of resemblance in structure. I think the difficulty there is that while we can talk about lots of things like doubles, and the use of doubles, which are very, very strong in fairy-tales. You can use... you can talk about repetition, you know the idea of three wishes, the idea of nine ravens, the idea of twelve dancing princesses. All the ways.....I think those have got a different relationship. They're not structural. That has to do with the whole tradition of oral story telling before print.

Melvyn Bragg : I think there is a great need for pattern, as you point out in your book, John, that **in the bible**, people will seek patterns to do with stories from .....taking certain words and patternising . Is the need for pattern something which they have in common? I mean in a sense, "counting your sheep"is putting a pattern on your sheep isn't it? And telling a story.....

Marina Warner : Well one of the ways that it's important to count and to make relationships in story telling is of

course to do with kinship systems. And that's very, very strong in the bible and very strong in early myths all over the world. I mean...

Melvyn Bragg : You're thinking of Leviticus?

Marina Warner : Yes, and of course, and in Leviticus there's also dietary prohibitions, which are also very important, because they position you with regard to nature. , and of course in society, and they mark you out. But those are stories that are again about telling..... making a network, making a pattern, but it doesn't mean that the story itself is structured. What worries me about the idea that there's embedded structure in stories that is mathematical, is that then it sends you back to some slightly fixed position, which is the sort of formalist position , that....or even perhaps, sometimes a union position, that you can't actually make new stories, that we're somehow hard-wired to be caught up in this kind of, you know, this world of solid... solid....

Melvyn Bragg : Well, actually, you don't think.....but John, John Allen Paulos thinks that the mathematicians can be *more* creative, *more* visionary, than novelists and playwrights , that's what you say in your book, one of the things you say in your book, and one presumes that there is a common imagination. You're not saying, Marina are you, that the imagination is sort of sliced up into little compartments and one can deal with fiction, but not having anything to do with mathematics? I should imagine... I'm talking to you now John, I should imagine that mathematics... er..mathematicians have to be as imaginative as anyone else who's trying to invent something?

John Allen Paulos : I couldn't agree more. I mean I think the attempt to divide the mind up into little parts, the literary part, the mathematical part is wrong-headed for want of a better term, and you're talking about patterns, I mean searching for patterns is basically what art and science is about, I mean we're little islands of order in this kind of (indistinct) thermodynamic sea of swirling static, and we have to search for whatever patterns are out there to stay alive, and whether we do it through art or through science....matters to some extent, in another sense it doesn't, I mean we need patterns to survive.

Melvyn Bragg : I'm just trying to hold to the similarities before we move on, because Galileo famously said, didn't he, "the language.....the book of life, I've discovered the secret of the book of life... it' is written... the book of life is written in the language of mathematics".....

John Allen Paulos : Right.

Melvyn Bragg : ... and he obviously used that analogy with a great deliberation, so what d'you make of that?

John Allen Paulos : I think that the world in some sense without subscribing to full-blown Platonism, I mean *is* mathematical, and er....

Melvyn Bragg : In what sense? In the Galileo sense, you mean?

John Allen Paulos : ... in the Galielean sense, and er....and so I mean, without as I say, I mean being a Platonist, there's a book written recently "The Number Sense" by Stanislaw Dahan (???) which has lots of nice items in it about the mathematical brain, Brian Butterworth wrote another book recently with the same kind of theme, but one of the contentions that Dahan makes is -I'm maybe mispronouncing his name - is that numbers are social constructs, that somehow they're in our heads, and that seems clearly not to be the case, that they have some sort of independent reality. Not just numbers, but numerical relations and relations in general, so....with.....so I'm not quite sure where I'm going with that, but....

Melvyn Bragg : D'you think that the number....sorry, Marina, you say what you want then I'll ask a question.

Marina Warner : No I was just going to say that the difficulty is that ... erm I 'm not going to comment on whether numbers are inherent or constructed...

John Allen Paulos : Right.

Marina Warner : ... but the difficulty is that any kind of use or application of them, especially outside the

mathematical field, I mean outside an equation, the minute you start applying them to stories, or even to anecdotes or to the kinds of stories you tell in your book which are not... you know which are to do with news items or.....

John Allen Paulos : Right.

Marina Warner : ....fears, panics, that take root....

John Allen Paulos : Right.

Marina Warner : ....that then you actually don't have the Platonic purity of the number, the number then does.....

John Allen Paulos : Oh of course, right.

Marina Warner : ... become a figure in this story telling, and I think that you can't use numbers to kind of, as it were, rectify the mistakes of stories. It seems to me that they are figures in the stories.

John Allen Paulos : Well I mean but, by numbers I mean I wasn't... I don't mean only numbers, I mean I think you know, I have too narrow a conception of what mathematics is. I think part of the problem is that the mathematics of stories, narrative logic, intentional logic, however you want to characterise this nebulously defined field, isn't well-developed yet. I mean people don't understand intentional logic very clearly, but....and er....but I think that people sense that there's something there that we can formalise the notion of situations, we can formalise the notion of structures, and that it just hasn't been done. I mean and there's desire *to* do it, and people often, I mean bring in mathematical metaphors in a kind of wrong-headed way, I mean there's a hoax in the US last year, a couple of years ago, this Physicist Alan Socal (???) at NYU.....

Marina Warner :Yes, yes, yes.

John Allen Paulos : .....I mean sent this article in as you know, this journal was full of scientific buzzwords, and it was accepted, it was total nonsense, so I mean there's a hunger for mathematical metaphor, scientific insight, but I mean **there's more of a will to employ them, than there is a knowledge about them**, which is still unformed.

Melvyn Bragg : Can I just... ? I want to stick to this....make the last point then we'll move on to our next section.

Marina Warner : Yes.

Melvyn Bragg : John has said that he sees mathematics can be just as visionary, in fact you claim sometimes *more* visionary and creative than novelists and dramatists. Now would you concede that that's a possibility, and if so what does that say about the two *apparently* different disciplines?

Marina Warner : Well I think actually he modifies that a bit in his book, you don't quite say that....

John Allen Paulos : . No.

Marina Warner : ....because you do agree that the prob.... one of the differences is that there is.....that maths is as it were..**not subjective**, you can't have a theorem coming out in two different ways
, but you know you can have a story coming out in two different ways, as indeed many great writers have done. You come at the same problem from different viewpoints
, and you get a different moral dilemma emerging from that, and of course any kind of drama, going back to the Greeks, but certainly Shakespeare will also go round that, so that, so that the mathematical equation.....

Melvyn Bragg : He doesn't modify it much, I've found the quotation, "Mathematics and scientific ideas frequently are more creative and visionary than novels or plays", that's pretty straightforward.

John Allen Paulos : But "frequently" allows the fact that frequently they're not! (laughter)

Marina Warner : And the law of probability, what is the probability here? Yes.

John Allen Paulos : Yeah!

Melvyn Bragg : Alright, alright, alright.

Marina Warner : No but I think that one of the..... I mean there is obviously a certain sort of **beauty and wonder** in these mathematical formulae which I don't understand . But I do think that the point of wonder and of fantasy is something that is very hard to factor in, if you're trying to see what is mathematical about stories.

Melvyn Bragg : I've heard mathematicians talking about their work, and talking about what's going on in their head while they're thinking about it, which is almost precisely the same as descriptions I've read of writers and artists what's going inside their heads, so where does that take us?

John Allen Paulos : I think that the problem is that the way mathematicians think about the subject when they're doing mathematics, which is full of little stories, pictures, arrows, kind of half-starts and so on is very different than what's presented. I mean once they've got the theorem they scrupulously cleanse it of any evidence that it was thought of by a human being, and it had a history or whatever. I mean it's often in a very rigorous way, I mean this is a definition, this a dilemma (??), here's the theorem, here's a corollary, and it's done elegantly, but.....

Melvyn Bragg : Rather like religious texts did in a way.

John Allen Paulos : Right.

Melvyn Bragg : Which are also sort of story lined. Can I ask you Marina Warner, early in this century as you know, Vladimir Prop (??) in 1922 wrote about... tried to reduce....thought he had reduced folk stories to almost mathematical structures.

Marina Warner : Yes.

Melvyn Bragg : The princess, in a folk story had a function, and nah, nah, nah, and he did... now what do you make of that?

Marina Warner : Well there were 31 functions.

Melvyn Bragg : Basic plots, yes.

Marina Warner : And 7 spheres of action, and this was in the 20s, and he....erm his book "The Morphology of the Folk Tale" was extremely influential after it was translated in the 60s, quite... or the 50s, late 50s into English, so it became... you know it had a kind of time lag of influence. But there were many other attempts, and what's interesting about these formalising attempts is of course that everybody came up with different numbers of functions, and different numbers of spheres of action, and different numbers of factors and things, and you do see, I mean the limits of that theory seem to me to be that you lose a sense of the context of response, not just the creator of the stories. You were talking about visions in the mind, of the mathematician and the writer being similar, it's to do with the context of response, and for instance Vladimir Prop himself says that the princess and the father can never be divorced, they function always together, but that's this clear effect of social conventions, in the society where, you know these stories developed, where the princess is given in marriage by her father and therefore the function's.....Now you would never say that a daughter and her father were the same function as any of us who have any children know (sniggers) this is no longer at all how society works.
So there you have a loss, a leeching out of something that I've always wanted to put back into the looking at myth, or fairy tale, or indeed other forms of fantastic writing, which is that one must see what function it is performing for the actual receivers, and those receivers change over time, and an example is something like the wicked step mother, who's been seen for a long time as an archetype, you know a figure of ....constantly recurring in Chinese...

Melvyn Bragg : Children are 100 times more liable to murder their step mother than their mother, yeah. Recent statistic, sorry.

Marina Warner : Really? Yes.

Melvyn Bragg : Yeah.

Marina Warner : Well the.... I mean that may have a great basis in experience, I mean one doesn't know...

Melvyn Bragg : Well we're coming on to statistics, we...(laughter) we're with a solemn statistician here, we've got to be careful! (laughter)

John Allen Paulos : Okay!

Marina Warner : Yes, but it seems to me that one mustn't forget that one of the reasons that there were so many step mothers, wicked or not, is that women....

John Allen Paulos : Fathers disappearing.

Melvyn Bragg : Or killed.

Marina Warner : ... yeh..but... and..but women were dying in childbirth, until the 19th century this was the main cause of female death, so the step mother was a very, very common figure in people's lives. I don't think it's just a psychoanalytical hostility towards you know, the "bad mother" that is expressed in the fairy tale.

Melvyn Bragg : Lets stick to numbers, er... erm....nevertheless these formalists said something, and I sort of ....maybe Joseph Campbell's "Hero of a thousand faces" took that up as well, and certainly when Umberto Eco did a breakdown of the Fleming novels, he said....er....the (clicks fingers) Bond novels , Ian Fleming, he said, "these seven things happened - Enter Bond. Enter villain. Bond and villain meet. Enter girl. Girl and durum dum dum....and ... now what d'you make of that attempt to relate? I mean because your also says, stories and mathematics diverge in certain ways, d'you think that is useful or not?

John Allen Paulos : I think it's useful, but I think again, the tools aren't in place. I mean to some extent it's a useful typology as long as you don't put too much weight on it.

I mean 7, 8 categories, 31, 33, I mean it's just an artefact and if people recognise it as such it's useful as far as it goes. I mean I think tools from recursive function theory, artificial intelligence where you analyse so-called scripts, like what you do when you go into a restaurant, what you do when you go to a movie, and so on, I mean is another way to get at the structure of stories, the structure of intentional logic. The fact is, I mean that there are kind of... fairly easily recognisable structures to stories to interchanges and ... but formalising this logic hasn't been done yet, there's something called situation.... er situation logic, where you try to formalise situations and I think that holds some promise, but er....again it's... and it's a laudable attempt, but er... it doesn't go very far and isn't all that impressive.

Marina Warner : One....

Melvyn Bragg : Errr... okay.

Marina Warner : One of the things I was puzzled by, and didn't know how you would express it or....

John Allen Paulos : Right.

Marina Warner : ..what you'd say about it, is what seems to me to be the defining characteristic of a lot of the fairy tale material, mythological material, and that is transformation, metamorphosis, that something changes into something else, the beast turns into a beautiful young man, the poor you know, peasant girl in the rags, turns into the princess, now that's very deep in....now how do you expre... how does one express that, because a number if.....

John Allen Paulos : No, I mean it's part of....

Marina Warner : ..if changed, is changed, whereas this is....

John Allen Paulos : It's part of what I was saying about extentionality , I mean in mathematics three is square root of nine....

Marina Warner : Yes.

John Allen Paulos : ..cube root of 27, it means the same thing, but in, you know, in every day life that's not the case. I mean Superman equals Clark Kent, but nevertheless, so you know, there's the story about... kid calls home and he asks for the cat, and his brother says, "The cat died" and he says "Ohh can't you break that to me more gently?", and he says, "Couldn't you have told me you know, 'The cats on the roof' and then you couldn't get it down, I call next time, and gradually in this way say that the cat died?", and the brother apologises for being so brusque, in telling him about his cats death. Anyway the guy says, "Well anyway how's mum?", and he says, "Well, she's on the roof", okay so I mean in 30 seconds, the meaning of "on the roof" changes, and there's no way at present to capture that in standard.

Melvyn Bragg : But in quantities in the wider mathematics, I mean we mustn't think of mathematics as multiplication tables....

John Allen Paulos : No, right.

Melvyn Bragg : ..things are changing in it all the time as well, aren't they?

John Allen Paulos : Errrr, but changing, I mean according to rules, I mean according to functions...

Melvyn Bragg : The rules change, and theories change.

John Allen Paulos : ... but er....

Melvyn Bragg : And our perception changes as a result of what we know. I mean Einstein "bent" Newton, and so on.

John Allen Paulos : . Yeah but still he bent Newton, but Newton is for all practical purposes, I mean unless you're really.....

Melvyn Bragg : So there's no relationship between mathematics and what Marina was proposing?

John Allen Paulos : Not in that sense, no.

Melvyn Bragg : Right. You talk about a relationship between mathematics and humour, and the structure of a joke, and we know from what we've read and what people have said, particularly comedians, writing comedians particularly, actually do talk very specific..... almost sometimes in engineering terms about how they pace, play, work out and deliver a joke, or a line of comedy, and we've got the great example of ....we can bring Lewis Carroll in here , all sorts of reasons, mathematician and errr... so where would, you say they exist on the same continuum, what d'you mean by that?

John Allen Paulos : Well you can think of both mathematics and humour as being forms of intellectual play, at different ends of the continuum, in mathematics more intellectual play, humour more playful, but in the middle are brain teasers, puzzles paradoxes. But I mean many jokes have a sort of quasi-mathematical structure, a lot of the same operations and structures that are common in mathematics, re-reversal, juxtaposition....

Melvyn Bragg : Such as?

John Allen Paulos : ..self reference. "Why do philosophers ask so many questions?", "Why shouldn't philosophers ask so many questions?" Reductio Ad Absurdum. Reductio Ad Absurdum is you know, used all the time in mathematics for the sake of disproving some assumption that whatever..... in humour is used for you know, the

pleasure of the Reductio. Self reference, these various syntactical operations that Lipo this French group is involved in, you know substituting the seventh word following a noun in some holy book and you get something funny or whatever, and then there's all... you know self reference, and in some sense the structure is the same as on-standard models, there's a notion in logic, I mean you go to a computer dating service, ask for somebody who's short, gregarious, likes formal clothes, cold weather, and the computer sends you a penguin (minor laughter), okay it's a non-standard model of these axioms, and in some sense the structure's the same as non-standard.....non-Euclidean geometry. What satisfies this, this, and that axiom of Euclidean geometry without the parallel postulate and you get this saddle- shaped surface (See Riemann/Lobochevsky), so the logic of many jokes is mathematical, quasi-mathematical, and er....

Marina Warner : But the effect of many jokes is also because of the community listening and their values.....

John Allen Paulos : Oh right of course.

Marina Warner : ... being overturned.

John Allen Paulos : I'm talking about the formal aspects, right.

Marina Warner : I know but the trouble is that if you take that kind of meaning out of it, that sort of social and you know, ethical meaning, you actually lose so much of the psychological and moral context.

John Allen Paulos : Oh yeah, but I'm not talking about that! (laughs) Of course, yeah good.

Marina Warner : No but that's the problem of actually saying that... because people attach themselves to these notions of structure, and then say, "well jokes always have to be like that"....

John Allen Paulos : Oh right I agree yeah right.

Marina Warner : ... and then you... but and then you get the target joke, the joke which makes the butt of someone and very often the pratfall joke is to do with actually demeaning someone else....

John Allen Paulos : Oh right, it's ... there's a psychology, an emotional.....

Marina Warner : ... and we have to think about that, and not, and not....yes.

John Allen Paulos : ... climate, is much different than the structure.

Melvyn Bragg : We've just got a few minutes, but let me dive right in. Lewis Carroll was a fine mathematician and he wrote "Alice in Wonderland"....

John Allen Paulos : Right.

Melvyn Bragg : ... a great work of imagination. Now does that, as it were, prove your case, or prove your case? First with you.

John Allen Paulos : Well I think.....I mean I like.....one thing about Carroll that's always struck me is that he seemed bothered and preoccupied, obsessed with the same kinds of misunderstandings of language, and language games as Wittgenstein

But given suitable hyperbole..... suitable allowance for hyperbole, that's what most people's mathematics education is like , it is "Uglification and derision", and they have this very narrow conception, where you know, which doesn't involve stories... which... they think of mathematics as very hierarchical and I think that these misconceptions are part of the reason people find it such a stultifying and distasteful subject.

Melvyn Bragg : Well not everybody finds it stultifying....

John Allen Paulos : No not everybody, but too many, too many.

Melvyn Bragg : ....I mean you're here... er Marina?

Marina Warner : Well I think that Carroll's very, very aware of how the adult world pressed on children and he took the child's part with Alice, and he actually gives the Red Queen, who is one of his tyrants, but he gives her the point that I would make about it, that he says, "What do you suppose is the use of a child without any meaning, even a joke should have some meaning, and a child's more important than a joke, I hope" (minor laughter). He was interested in the child as a person, and was worried that these logical structures that he played with so brilliantly and which were to do with jokes and puzzles and games, did actually somehow lose the sense of the person in the child

John Allen Paulos : Ohhh, I think that.....there's a joke that's not very funny, but is relevant, two hard headed scientists are talking and they're saying, "Well, why don't we do away with these notion... narrative notion stories. I hate that stuff, let's just stick to facts and numbers", and the other one says, "I couldn't agree more, let's just resolve mathematical facts, scientific facts", and so on, and the joke such as it is, is that they're using these notions of hate and clarity and so mathematics presupposes and grows out of these murky notions of story telling, narrative and so on.

Melvyn Bragg : Thank you very much. That was a gallop wasn't it? But I enjoyed it, I hope you did. That's John Allen Paulos and Marina Warner and thank you for listening.